

TRIGONOMETRY

E6.3 Exact trigonometric values

Know the exact values of:

1 $\sin x$ and $\cos x$ for $x = 0^\circ, 30^\circ, 45^\circ, 60^\circ$ and 90° .

2 $\tan x$ for $x = 0^\circ, 30^\circ, 45^\circ$ and 60° .

MEMORIZE THE VALUES

	0	30	45	60	90
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined. ∞

Trick to make table :

sin 0 30 45 60 90

$$\sqrt{\frac{0}{4}} = 0 \quad \sqrt{\frac{1}{4}} = \frac{1}{2} \quad \sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}} \quad \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2} \quad \sqrt{\frac{4}{4}} = 1$$

cos 0 30 45 60 90

$$\sqrt{\frac{4}{4}} = 1 \quad \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2} \quad \sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}} \quad \sqrt{\frac{1}{4}} = \frac{1}{2} \quad \sqrt{\frac{0}{4}} = 0$$

$\frac{\sin}{\cos} = \tan$

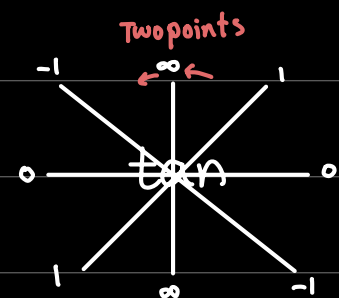
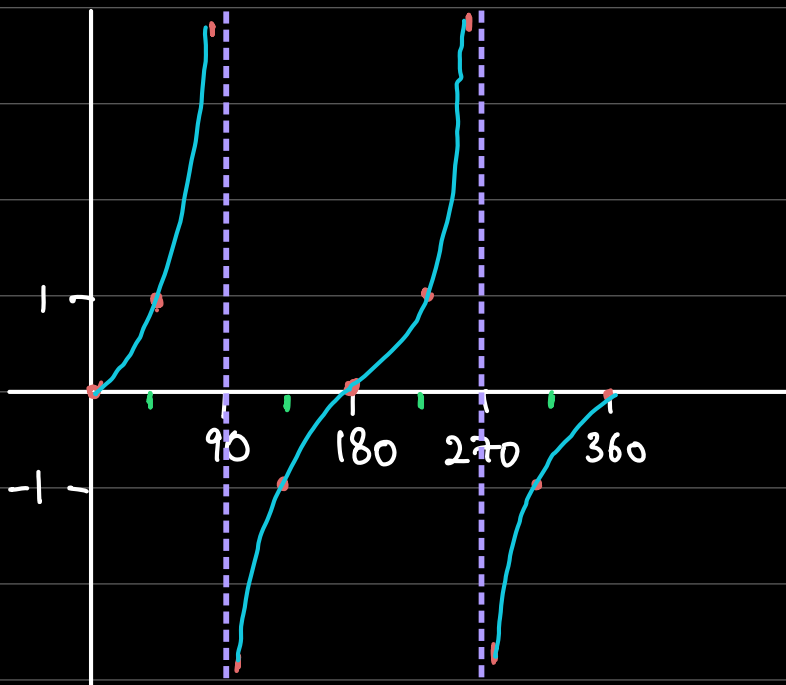
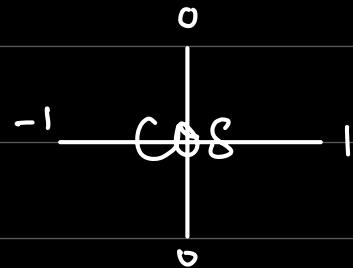
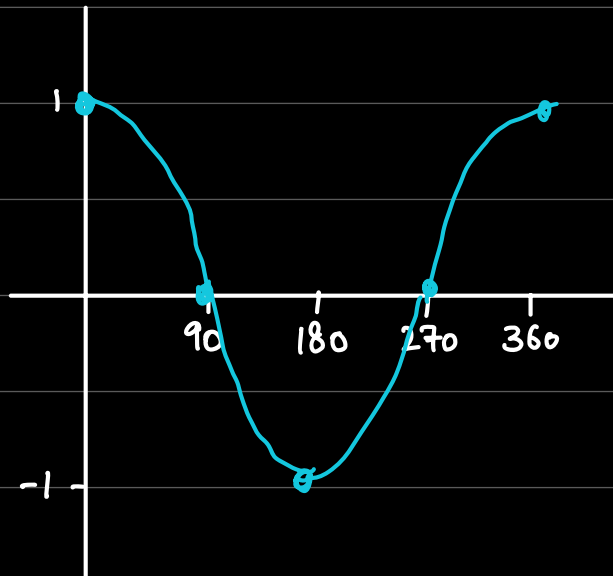
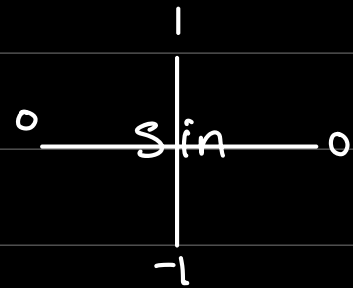
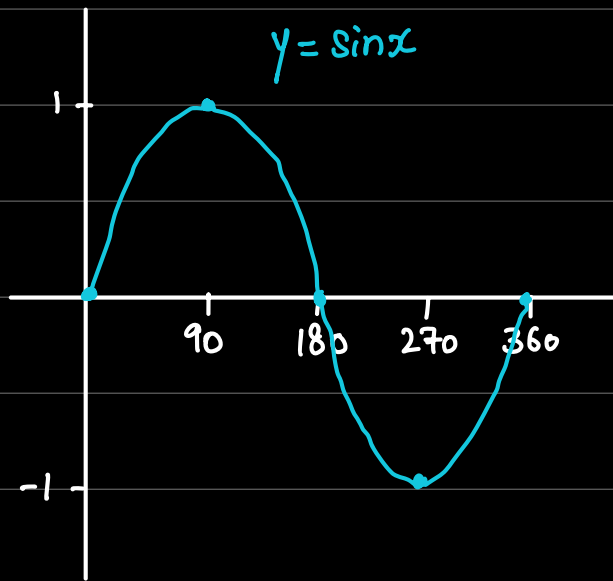
$$0 \div 1 = 0 \quad \frac{1}{2} \div \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}} \quad \frac{1}{\sqrt{2}} \div \frac{1}{\sqrt{2}} = 1 \quad \frac{\sqrt{3}}{2} \div \frac{1}{2} = \sqrt{3} \quad 1 \div 0 = \text{undefined} \quad \infty$$

GRAPHS

E6.4 Trigonometric functions

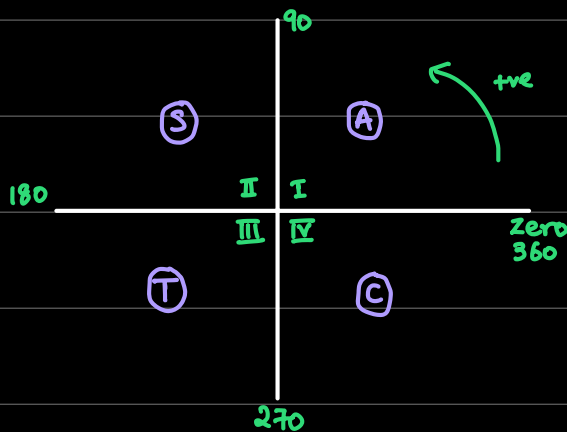
1 Recognise, sketch and interpret the following graphs for $0^\circ \leq x \leq 360^\circ$:

- $y = \sin x$
- $y = \cos x$
- $y = \tan x$.



EQUATION SOLVING

This is some part of a complete concept you will study in A-Levels.



Step 1: Take inverse ignoring all $\pm$ signs and find α .

Step 2: Decide Quadrant and then apply this:

$x = 180 - \alpha$	$x = \alpha$
$x = 180 + \alpha$	$x = 360 - \alpha$

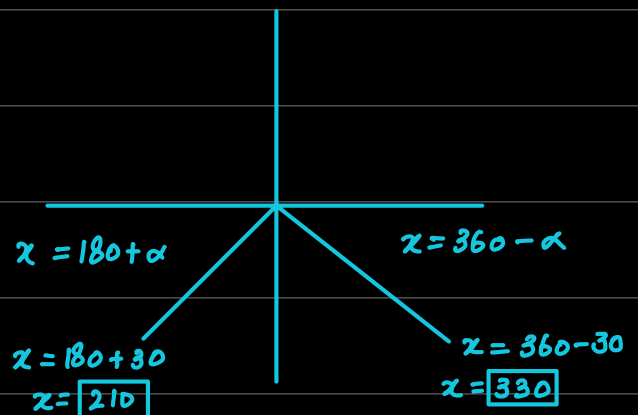
1) $\sin x = \frac{-1}{2}$
 QUADRANT

$$0 \leq x \leq 360$$

$$\alpha = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\alpha = 30$$

$$x = 210, 330$$



2-

$$\tan x = -\sqrt{3}$$

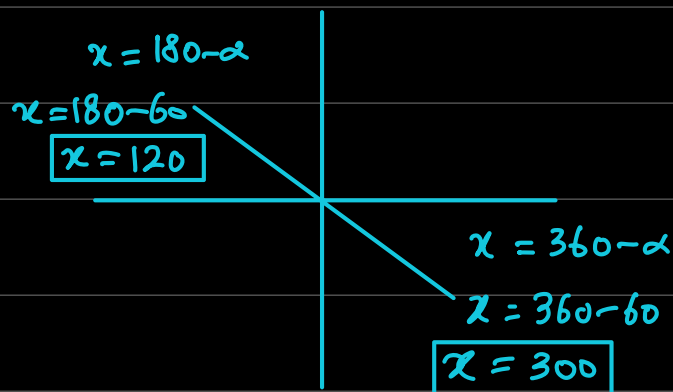
Quadrant (2 and 4)

$$0 \leq x \leq 360$$

$$\alpha = \tan^{-1}(\sqrt{3})$$

$$\alpha = 60$$

$$x = 120, 300$$

19 Solve the equation $2 + 5 \cos x = 0$ for $0^\circ \leq x \leq 360^\circ$.

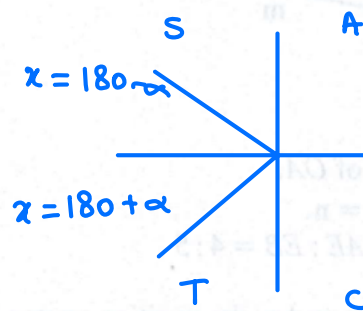
$$5 \cos x = -2$$

$$\cos x = -\frac{2}{5}$$

Quadrant (2, 4)

$$\alpha = \cos^{-1}\left(\frac{2}{5}\right)$$

$$\alpha = 66.42$$

CALCULATOR
ALLOWED

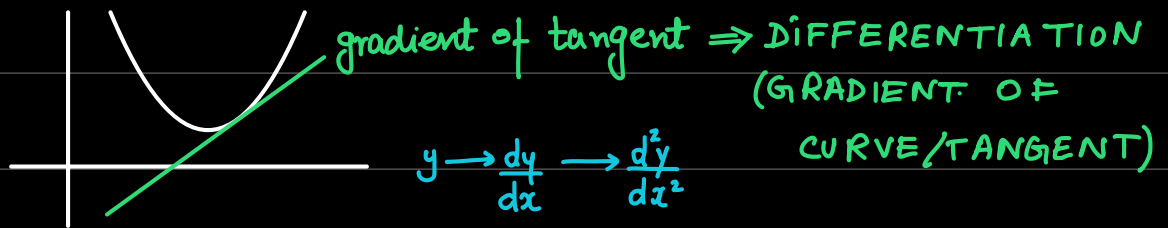
$$x = 180 - \alpha = 180 - 66.42 = \boxed{113.58}$$

$$x = 180 + \alpha = 180 + 66.42 = \boxed{246.42}$$

 $x = \dots\dots\dots$ or $x = \dots\dots\dots$ [3]

DIFFERENTIATION

This is some part of a complete concept you will study in A-Levels.



BASIC RULES:

1) $x \rightarrow 1$

$2x \rightarrow 2$

$5x \rightarrow 5$

$-3x \rightarrow -3$

2) Alone number $\rightarrow 0$
without x -term

$5 \rightarrow 0$

$-2 \rightarrow 0$

$3 \rightarrow 0$

MAIN RULE:

STEP 1: BRING POWER BACK AND MULTIPLY

STEP 2: SUBTRACT ONE FROM POWER.

EXAMPLES:

a) $y = 12x^4$

$$\frac{dy}{dx} = 12(4)x^3$$

$$\frac{dy}{dx} = 48x^3$$

2) $y = x^4 - 3$

$$\frac{dy}{dx} = 4x^3 - 0$$

$$\frac{dy}{dx} = 4x^3$$

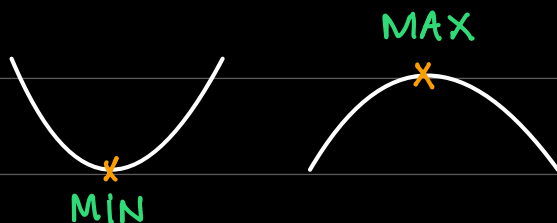
$$1b) \quad y = 5x^3 - 3x^2 + 12x + 5$$

$$\frac{dy}{dx} = 5(3)x^2 - 3(2)x + 12 + 0$$

$$\frac{dy}{dx} = 15x^2 - 6x + 12$$

STATIONARY POINT AND NATURE

↓
TURNING POINT



$\frac{dy}{dx} = 0$ gives you stationary points.

$\frac{d^2y}{dx^2}$ decides whether it is a minimum nature or a maximum nature turning point.

$\frac{d^2y}{dx^2}$ $\begin{cases} \rightarrow \oplus \text{ Min } \cup \\ \rightarrow \ominus \text{ Max } \cap \end{cases}$

$y \xrightarrow{\text{DIFF}} \frac{dy}{dx} \xrightarrow{\text{DIFF}} \frac{d^2y}{dx^2} \begin{cases} \oplus \text{ Min } \cup \\ \ominus \text{ Max } \cap \end{cases}$

↓
 $\frac{dy}{dx} = 0$

Tells you the stationary points.

NOT FOR OL (4024)
ONLY FOR IGCSE PEOPLE

22 A curve has equation $y = x^3 + x^2 - x$.

The curve has a stationary point at $\left(\frac{1}{3}, -\frac{5}{27}\right)$.

NON
CALCULATOR
QUESTION

(a) Find the coordinates of the other stationary point.

$$y = x^3 + x^2 - x$$

$$\frac{dy}{dx} = 3x^2 + 2x - 1$$

$$\frac{dy}{dx} = 3x^2 + 2x - 1$$

$$\frac{dy}{dx} = 0$$

$$3x^2 + 2x - 1 = 0$$

$$3x^2 + 3x - x - 1 = 0$$

$$3x(x+1) - 1(x+1) = 0$$

$$(x+1)(3x-1) = 0$$

$$x+1 = 0$$

$$x = -1$$

$$, 3x-1 = 0$$

$$x = \frac{1}{3}$$

We already know
this one.

$$y = x^3 + x^2 - x$$

$$= (-1)^3 + (-1)^2 - (-1)$$

$$= -1 + 1 + 1$$

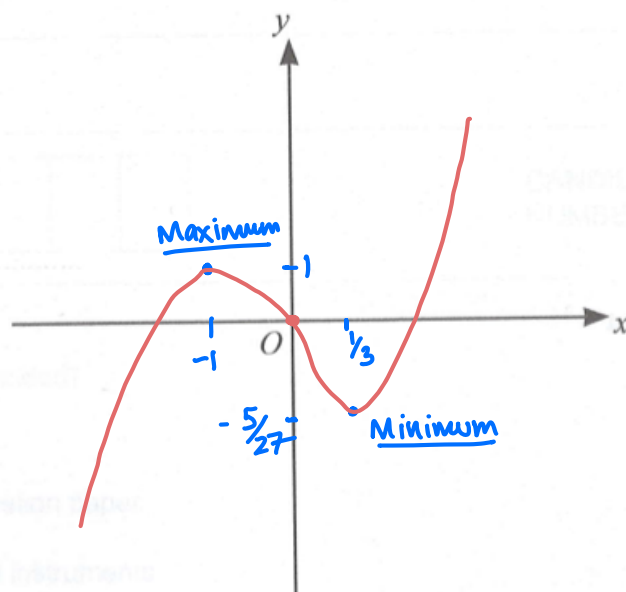
$$y = 1$$

(-1, 1) [5]

(b) By sketching the graph of $y = x^3 + x^2 - x$, determine whether the stationary point

$\left(\frac{1}{3}, -\frac{5}{27}\right)$ is a maximum or a minimum. $x=0, y=0$

$(-1, 1)$



$\left(\frac{1}{3}, -\frac{5}{27}\right)$ is a Minimum. [2]

(c) The equation $x^3 + x^2 - x = k$ has fewer than 3 solutions.

Find the range of possible values for k .

$y = k$ (Horizontal)

$$k \geq 1$$

$$k \leq -\frac{5}{27}$$

..... [2]

20

$$y = 2x^k + ux^7 \quad \text{and} \quad \frac{dy}{dx} = 18x^{k-1} + 21x^6$$

Find the value of k and the value of u .

$$y = 2x^k + ux^7$$

$$\frac{dy}{dx} = 2kx^{k-1} + u(7)x^6$$

$$\frac{dy}{dx} = 18x^{k-1} + 21x^6$$

$$2k = 18$$

$$k = 9$$

$$7u = 21$$

$$u = 3$$

$k =$

$u =$

[2]

STEM AND LEAF DIAGRAMS

we get rid of repeats.

1.17	1.21	1.31	1.43
1.18	1.27	1.32	1.47
1.19	1.23	1.37	1.49

must be integer

↑
Stem

Leaf

must be single digit.

(last significant figure generally)

11	7	8	9 ^{3rd}	} $n=12$
12	1 ^{4th}	2	3 ^{6th}	
13	1 ^{7th}	2	7 ^{9th}	
14	3 ^{10th}	7	9	

key: 11 | 7
means
1.17

$$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{ term} = \left(\frac{12+1}{2}\right)^{\text{th}} \text{ term} = 6.5^{\text{th}} \text{ term} \left\{ \begin{array}{l} 6^{\text{th}} \Rightarrow 1.23 \\ 7^{\text{th}} \Rightarrow 1.31 \end{array} \right\} \frac{1.23 + 1.31}{2} = \text{Median} = 1.27$$

$$\text{Lower Quartile} = \left(\frac{n+1}{4}\right)^{\text{th}} \text{ term} = \left(\frac{12+1}{4}\right)^{\text{th}} \text{ term} = 3.25^{\text{th}} \text{ term} \left\{ \begin{array}{l} 3^{\text{rd}} \Rightarrow 1.19 \\ 4^{\text{th}} \Rightarrow 1.21 \end{array} \right\} \frac{1.19 + 1.21}{2} = \text{LQ} = 1.2$$

$$\text{Upper Quartile} = 3 \left(\frac{n+1}{4}\right)^{\text{th}} \text{ term} = 3 \left(\frac{12+1}{4}\right)^{\text{th}} \text{ term} = 9.75^{\text{th}} \text{ term} \left\{ \begin{array}{l} 9^{\text{th}} \Rightarrow 1.37 \\ 10^{\text{th}} \Rightarrow 1.43 \end{array} \right\} \frac{1.37 + 1.43}{2} = \text{UQ} = 1.4$$

$$\begin{aligned} \text{Interquartile Range} &= \text{UQ} - \text{LQ} \\ &= 1.4 - 1.2 = 0.2 \end{aligned}$$

- 26 The pulse rates, in beats per minute, of a random sample of 15 small animals are shown in the following table.

115	120	158	132	125
104	142	160	145	104
162	117	109	124	134

(i) Draw a stem-and-leaf diagram to represent the data.

[3]

(ii) Find the median and the quartiles.

[2]

10	4, 4, 9
11	^{4th} <u>5</u> , 7
12	0, 4, ^{8th} <u>5</u>
13	2, 4
14	2, ^{12th} <u>5</u>
15	8
16	0, 2

Key:

12 | 4 means

124 beats
per min.

$$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{ term} = \left(\frac{15+1}{2}\right)^{\text{th}} \text{ term} = 8^{\text{th}} \text{ term} \Rightarrow \boxed{\text{Median} = 125}$$

$$LQ = \left(\frac{n+1}{4}\right)^{\text{th}} \text{ term} = \left(\frac{15+1}{4}\right)^{\text{th}} \text{ term} = 4^{\text{th}} \text{ term} \Rightarrow \boxed{LQ = 115}$$

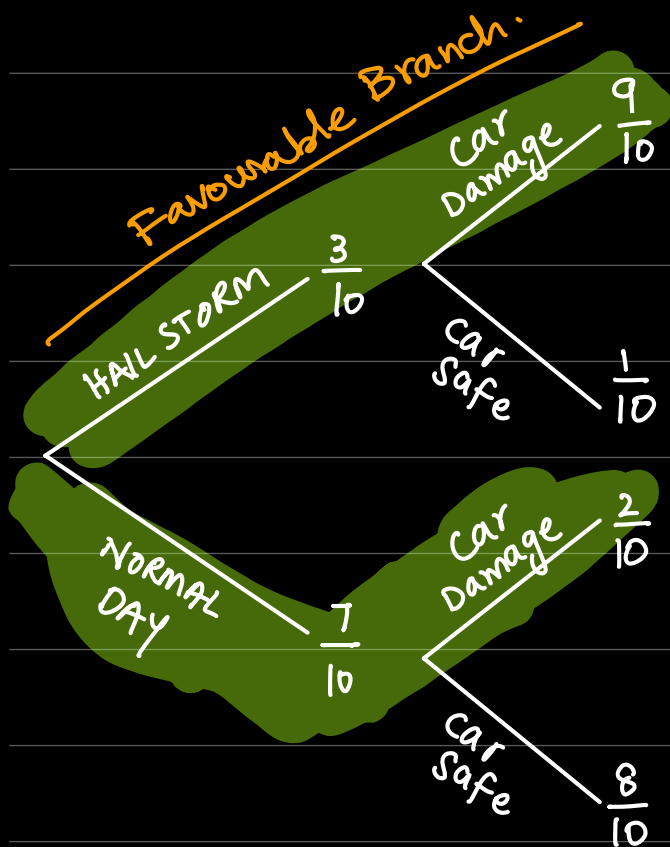
$$UQ = 3 \left(\frac{n+1}{4}\right)^{\text{th}} \text{ term} = 3 \left(\frac{15+1}{4}\right)^{\text{th}} \text{ term} = 12^{\text{th}} \text{ term} \Rightarrow \boxed{UQ = 145}$$

$$\begin{aligned} \text{Interquartile range} &= UQ - LQ \\ &= 145 - 115 = 30 \end{aligned}$$

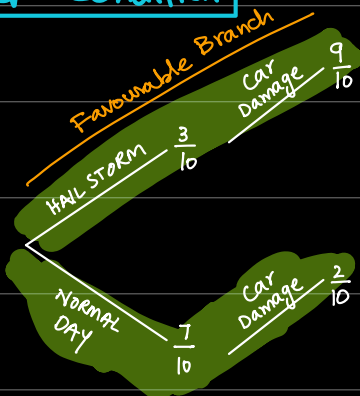
CONDITIONAL PROBABILITY

YOU WILL HAVE TWO EVENTS: $A = \text{FIRST}$
 $B = \text{SECOND}$

GIVEN THAT SECOND EVENT HAS ALREADY HAPPENED,
FIND PROBABILITY OF FIRST EVENT



After Condition:



Q: It is given that a car was damaged, find the probability that there was a hailstorm that day?

Now tree diagram has only two branches available.

Find your favourable branch.

$$\begin{aligned} P(\text{event}) &= \frac{\text{Favourable}}{\text{Total}} \\ &= \frac{\frac{3}{10} \times \frac{9}{10}}{\left(\frac{3}{10} \times \frac{9}{10}\right) + \left(\frac{7}{10} \times \frac{2}{10}\right)} \\ &= \frac{\frac{27}{100}}{\frac{27}{100} + \frac{14}{100}} \\ &= \frac{\frac{27}{100}}{\frac{41}{100}} = \frac{27}{41} \end{aligned}$$

Knowledge of notation, $P(A|B)$, and formulas relating to conditional probability is **not** required.